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# **FATIGUE LIFE VARIABILITY IN LARGE ALUMINUM FORGINGS WITH RESIDUAL STRESS**

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# Fatigue Life Variability in Large Aluminum Forgings with Residual Stress

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As part of a larger residual stress modeling and standardization program, a computational assessment of the variability in fatigue crack growth life that can result from variability in residual stresses has been conducted. A detailed finite element analysis of the forge/quench/coldwork/machine process was performed in order to predict the bulk residual stresses in a fictitious aluminum bulkhead. The residual stress profiles on ten critical planes were used to calculate residual stress intensity factors for a range of crack types that are typical for aircraft structural details. These stress intensity factors were used in a standard, linear-elastic-fracture-mechanics-based fatigue crack growth algorithm in order to predict fatigue life for typical fighter aircraft spectrum loading. Calculations were made for residual stress profiles which reflected variability due to machined part placement within the parent forging. For the conditions considered in this study (forging process, machined part placement, critical location / crack geometry, material and fatigue spectrum) we have found that variability in fatigue crack growth life due to part placement is on the order of +/- 20% about the mean. Furthermore, we have found from scaling the residual stress distributions for these conditions, that fatigue crack growth life sensitivity is on the order of 15% to 50% change in crack growth life with every 5ksi change in peak residual stress.

## I. Introduction

The fully effective utilization of large, thick metallic product forms in aerospace structures has been hampered in the past by inadequate understanding of, and sometimes inaccurate representation of, bulk residual stresses and their impact on design mechanical properties. In recent years, significant advances in both computational and experimental methods have led to vastly improved characterization of residual stresses in large, forged structural components. As a result, new design approaches which require the formal inclusion of residual stresses in the design analysis, have been enabled. In particular, the impact of residual stresses on durability and damage tolerance can now be assessed, and more importantly, accounted for at the beginning of the design cycle.

In an effort to support the development of this next-generation design capability, the AFRL sponsored Metals Affordability Initiative (MAI) consortium<sup>1</sup> has recently concluded research on the standardization of residual stress modeling and measurement techniques. This paper summarizes one portion of that program: a computational assessment of the variability in fatigue crack growth life that can result from variability in residual stresses.

## II. Residual Stresses in a Representative Bulkhead

In response to both internal requirements (for forging process design) and external requirements (customer requirements for forging mechanical property and residual stress data), Alcoa has, and continues to develop the capability for computational simulation of the forge, quench, cold work and machining processes. In order to handle the complex, 3D shapes typical of large aircraft forgings, Alcoa has developed its own automatic 100% hex element

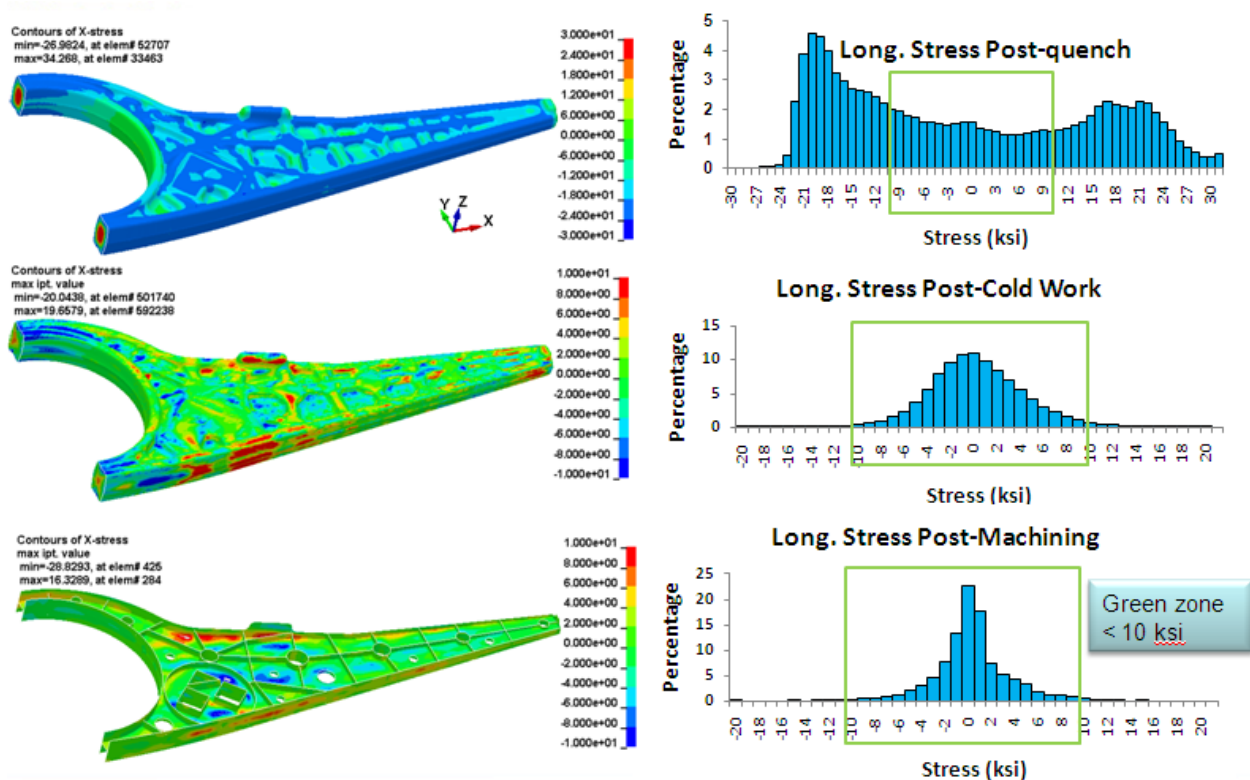
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mesher targeted for the modeling of heat-treatment quench using the finite-element code LS-Dyna (from Livermore Software Technology Corporation) <sup>2</sup>. Alcoa has invested in developing proprietary material models for aluminum alloys. For the rapid cooling of a quench analysis Alcoa uses a material model that can handle a large temperature range with small strain and strain rate dependencies. An entirely unique and different material model is used for the room temperature simulation of cold work stress relief that is dependent on the hours of post-quench natural aging history and the cooling rate history. The LS-Dyna solver is the central component of Alcoa's integrated software environment. This tool suite includes other commercial and proprietary codes and has been used by the Alcoa forging business unit to improve the forging process by designing and optimizing for consistent low residual stress..

In support of the current MAI program, as well as related programs requiring a non-proprietary structural configuration, Alcoa modeled the forge, quench, cold-work and machining of a fictitious, but representative, aircraft bulkhead. As shown in Figure 1, the simulation indicates that virtually all of the residual stresses in the finished part fall within the +/- 10 ksi "green zone."



**Figure 1. Calculated Residual Stresses in Alcoa Bulkhead.**

Results were generated for three machined part configurations. In the first, the machined part was at the optimum placement within the forging. In the other two, the machined part was positioned 0.3 inch above and 0.3 inch below the nominal position. Critical locations within each bulkhead were selected based on geometric, service (fatigue) loading, and residual stress considerations. Five critical planes (section cuts) were defined for each bulkhead; a typical one of which is shown in Figure 2.

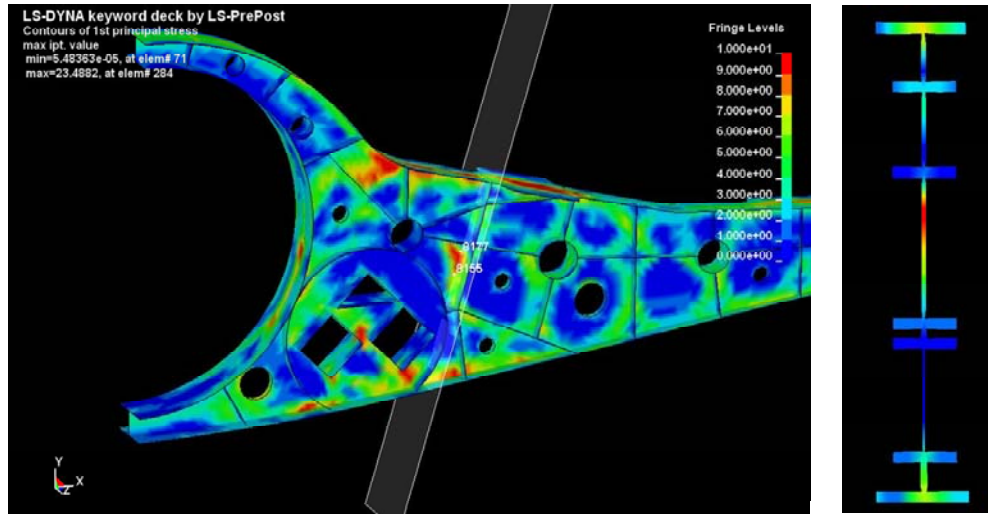


Figure 2. Typical (section 4 of 5) Critical Plane – Wing Root.

Residual stress profiles were determined at each critical plane by taking a cross-section at that location and determining the stresses on the cutting plane. Two types of profiles were generated in each case. In the first, the maximum principal stress at each recovery point was extracted and used to define the RS profile, regardless of the direction of the principal stress axes at that point. In the second, the stresses were rotated at each recovery point to determine the stress acting normal to the cutting plane. Since the maximum principal stress profile on a given plane will always be as or more conservative than the normal stress profile, the max principal profiles were used in this study.

Two regions from each cross-section, a web and a flange, were evaluated. The impact that part placement within the forging can have on residual stress was assessed by comparing profiles from each of the three bulkheads (bh1 = nominal part placement, bh2 = 0.3 in. below nominal, bh3 = 0.3 in. above nominal). As shown in Figure 3, part placement had an impact on both the relative position and the peak of the residual stress distribution, and this was true for both the web and the flange. A summary (histogram) of the peak residual stresses at the ten critical locations in each of the three bulkheads is shown in Figure 4.

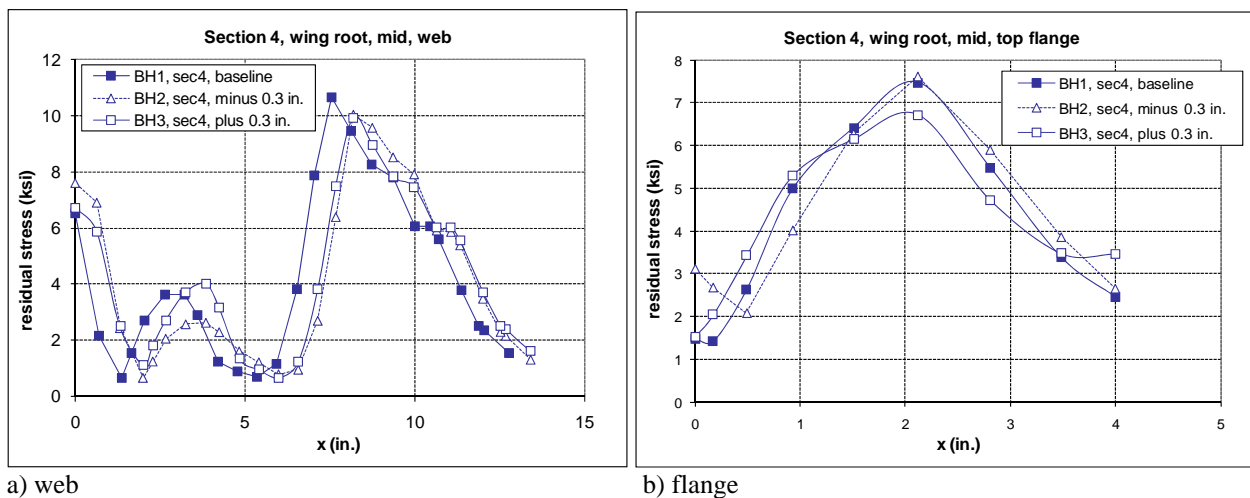
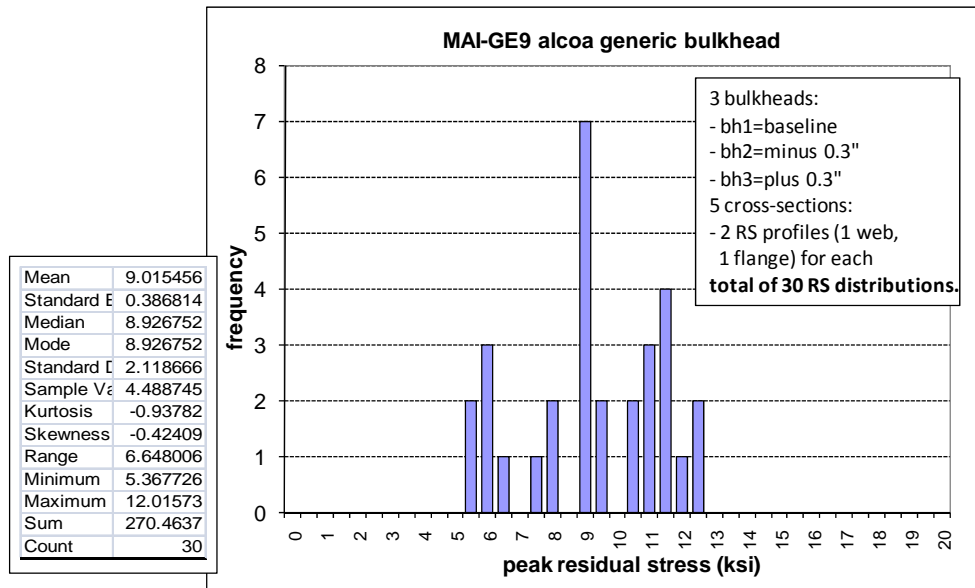


Figure 3. Residual Stress Profiles in Web and Flange of Section 4.



**Figure 4. Summary of Peak Residual Stresses at Ten Selected Critical Locations.**

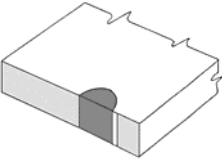
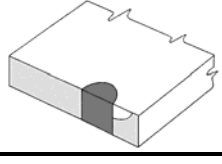
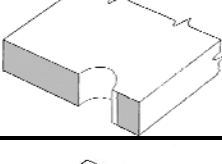
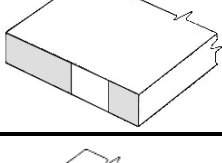
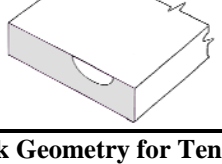
### III. Fatigue Crack Growth Analysis

Fatigue crack growth (FCG) analyses, in which the influence of residual stress was modeled using a residual stress intensity factor, were conducted. The control point (CP) geometry for each analysis was defined based on dimensions from the bulkhead section cut (segment thicknesses was based on shell element thicknesses). The crack configuration was based on 1 of 5 control point models as shown in Figure 5. The 'PA' models were used to simulate cracks in web-to-stiffener radii, the 'FA' models were used to simulate cracks at loaded fastener holes in the bulkhead cap flanges, and the 'HA' model was used to simulate a crack at the penetration in the section 5 web.

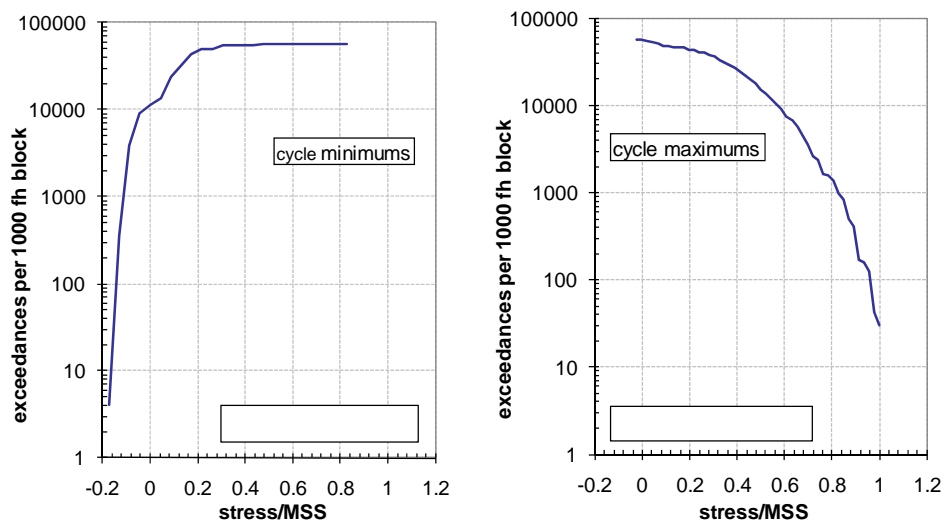
In this study, the standard, LEFM-based approach for the analysis of fatigue crack growth in the presence of residual stresses<sup>3</sup> was used. In this approach, the initial bulk residual stress field in the uncracked body is assumed to be unchanged during fatigue crack growth. With the additional assumption that the bulk material response is elastic, both stresses and stress intensity factors may be superimposed. In particular, the stress intensity factor due to the residual stress field and those due to the applied cyclic stresses may be summed to determine the appropriate total stress intensity factor at each cycle of the analysis. The stress intensity factor (SIF) due to the residual stress was calculated using the Green's function approach; 1D solutions<sup>4</sup> were used for cracks in thin webs and flanges while 2D solutions<sup>5</sup> were used for part-through cracks in thick sections. The total SIF was used to calculate fatigue crack growth rate which in turn was integrated to obtain the crack growth life. The details of LEFM-based fatigue crack growth analysis are described elsewhere<sup>6</sup>. Repeated simulations of fatigue crack growth life were run using the residual stress profiles described above. Fatigue crack growth lives were calculated at each critical location using standard damage tolerance initial flaw sizes<sup>7</sup> and a fighter aircraft wing bending moment (WBM) spectrum (tension dominated). See Figure. 6.

### IV. Calculated Fatigue Life Variability due to Part Placement

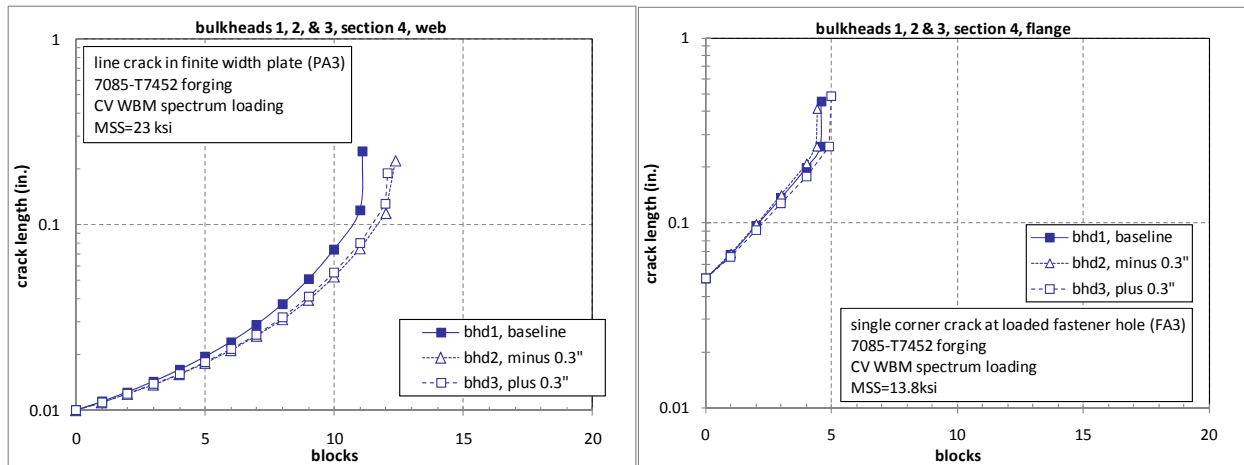
The impact that the variability in residual stresses due to part placement has on calculated FCG life for the flange and web locations on cross-section 4 is shown in Figure 7. Similar results were obtained for each of the other cross-sections. Examination of the results indicated that the amount of variability due to part placement within the forging to be expected in calculated crack growth lives is dependent on the nature of the residual stress profile and on the fracture model. For example, in section 4 (Figure 7) we see about an 11% variation in life for cracking in the web, and about a 12% variation in life for cracking in the flange. Note that this variation occurs due to differences in both the shape of the RS profile and the peak value. For sections with little variation in the RS profile, as expected, there is minimal variation in the calculated FCG life. On the other hand, there is significant variation in the RS profiles, then significant variation in FCG life results. The results are summarized for all five cross-sections in Table 1.

locations		code	description
section 1, flange section 2, flange		FA1	one through thickness crack
section 4, flange		FA3	one quarter-elliptical corner crack
section 5, web		HA1	one through thickness crack
section 1, web section 2, web section 3, web section 4, web		PA3	eccentric through thickness crack
section 3, flange section 5, flange		PA6	semi-elliptical surface crack

**Figure 5. Assumed Crack Geometry for Ten Critical Locations.**



**Figure 6. Exceedance Diagrams for Wing Bending Moment Spectrum.**



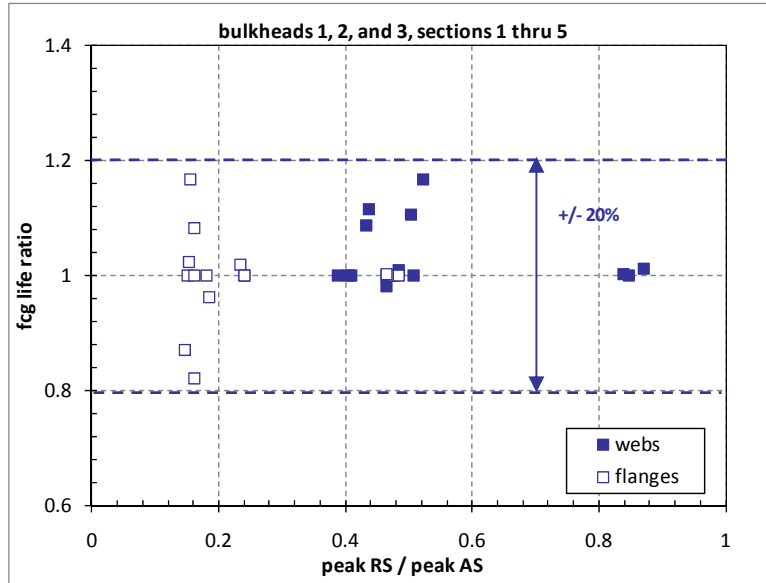
**Figure 7. Calculated Dependence of FCG Life on RS Variability due to Part Placement, Section 4, Web and Flange.**

**Table 1. Calculated FCG Life Variability due to RS Variability due to Machined Part Placement Within Forging.**

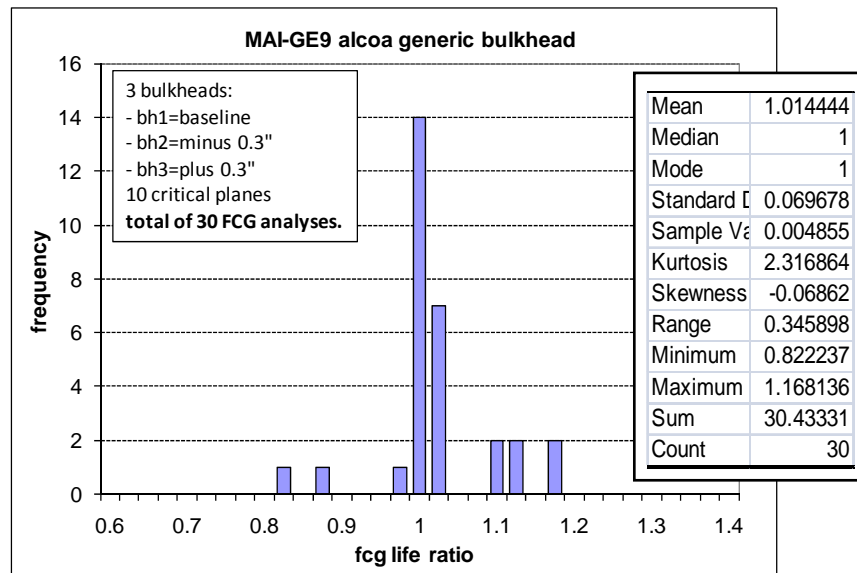
location	min life ratio	max life ratio	variability, $\Delta\text{life}(\%)$
section 1, flange	0.870	1.024	15.4
section 1, web	1.000	1.167	16.7
section 2, flange	0.822	1.168	34.6
section 2, web	0.999	1.000	0.1
section 3, flange	1.000	1.002	0.2
section 3, web	0.981	1.009	2.8
section 4, flange	0.962	1.084	12.2
section 4, web	1.000	1.115	11.5
section 5, flange	1.000	1.020	2.0
section 5, web	1.000	1.012	1.2

Note that these results are specific to this material and spectrum. We can assume that for general structural components, there will be many different details or features, that any residual stresses that might be present will be in various positions and orientations with respect to these features, that the operating stresses in the part will be concentrated to differing degrees by the features, that the concentrated stresses will interact differently with the bulk RS depending on position and orientation, and finally, that different crack configurations can be expected at different types of features. This level of potential variability in the factors that strongly influence fatigue life might initially lead one to believe that there is little hope for formal RS management. However, as discussed above, various crack configurations at a number of typical structural details, with different magnitudes of applied spectrum loading were studied here and still the variability in calculated FCG life was less than 40%. See Figures 8 and 9. This falls well within the ‘scatter factor of two’ requirement that is typical for structure that is designed and operated according to fracture mechanics based damage tolerance principles. Furthermore, there may be opportunity to draw some general conclusions regarding variability as more combinations of RS, structural detail and crack configuration are studied.





**Figure 8. Calculated variability in FCG life due to machined part placement within forging.**



**Figure 9. Calculated variability in FCG life due to machined part placement within forging.**

## V. Fatigue Life Sensitivity Study

In addition to machined part placement, there are a variety of other process parameters that can strongly influence bulk residual stress and, therefore, act as a source for residual stress variability. These parameters include quench rates, cold work levels and aging time and temperature. Direct calculation of the variability in RS due to such parameters was outside the scope of the current program. Furthermore, there is a position dependent variability in the RS profiles that ultimately drive crack formation and growth, due to the statistical nature of initial flaw site locations. So, in lieu of a variability study based on directly calculated bulk residual stresses, a sensitivity study, based on the RS profiles from the previous section was performed. The purpose of the sensitivity study was to quantify the dependence of fatigue crack growth life on bulk RS field characteristics (notably peak RS) for a set of assumed bulk RS profiles (namely the ones generated for the machined part placement study). Knowing this

sensitivity allows analysts / designers to understand the potential impact of bulk residual stresses on the strength and life of forged parts.

In order to determine fatigue crack growth life sensitivity, a series of nine RS profiles at each critical plane was generated by scaling the baseline profile. The following scale factors were used: 0.8, 0.9, 0.95, 0.98, 1.0, 1.02, 1.05, 1.1 and 1.2 (resulting in 270 profiles). The results for critical plane 4 of bulkhead 1 (nominal placement) are shown in Figure 10. Similar data were generated for the other critical planes and for bulkheads 2 and 3; a summary of the distribution of peak stresses is shown in Figure 11.

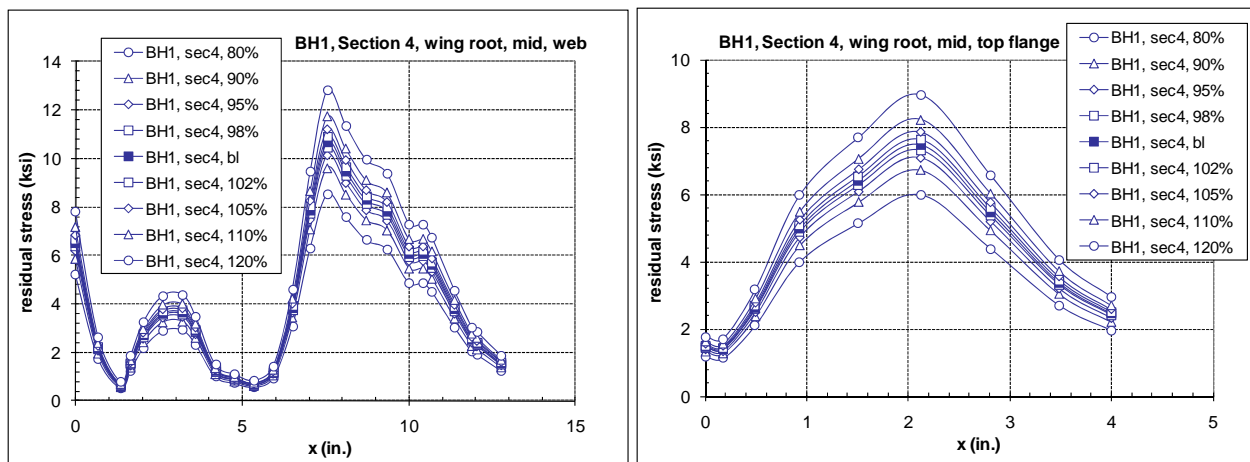


Figure 10. Scaled Residual Stress Profiles in Web and Flange of Bulkhead 1, Section 4.

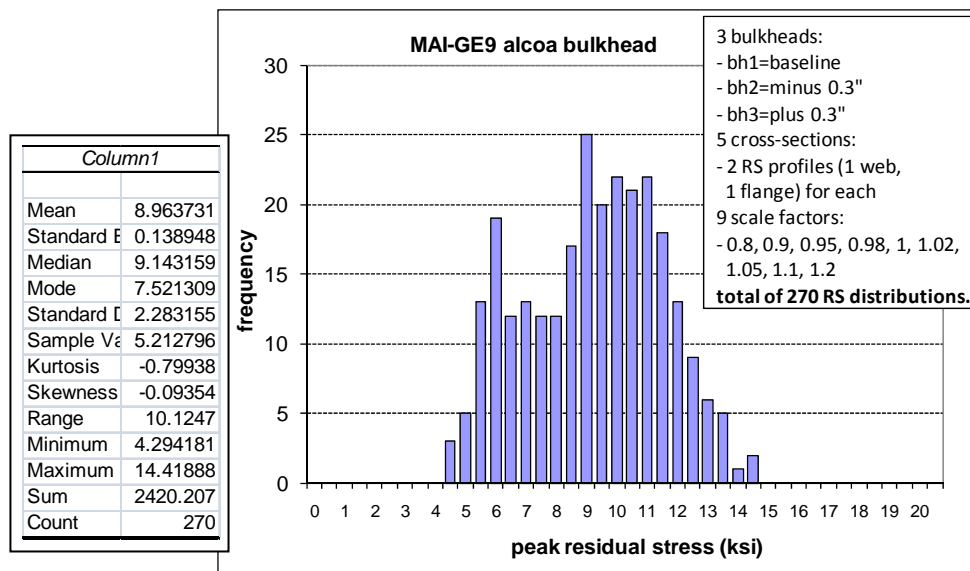
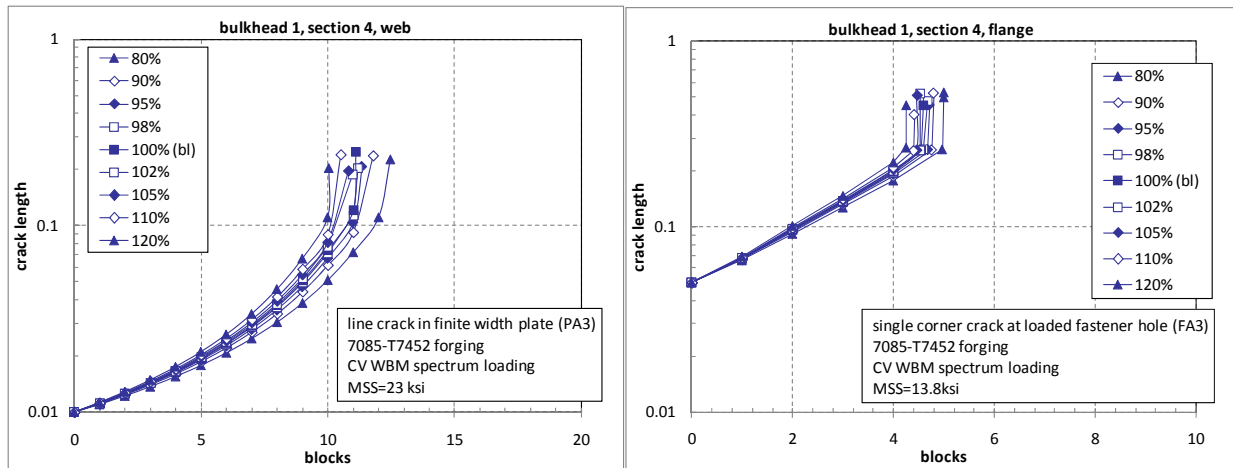


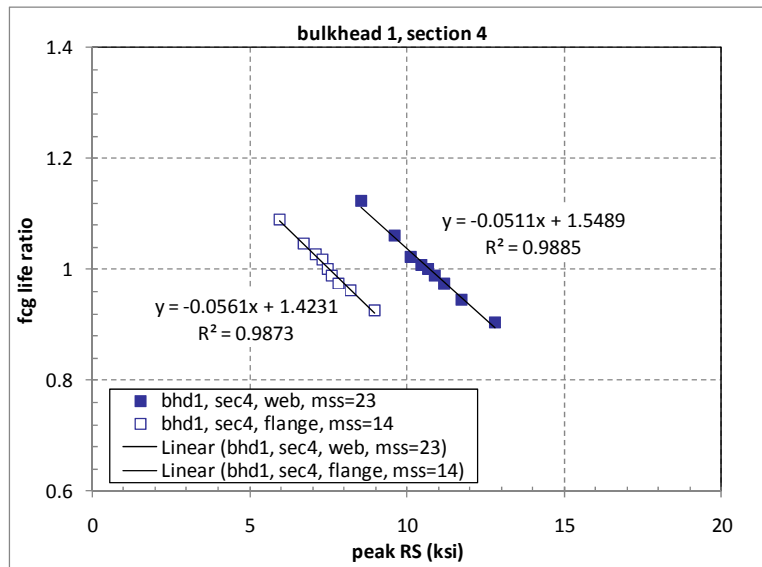
Figure 11. Summary of Peak Residual Stresses for Scaled Profiles.

Using the input data / assumptions discussed above, along with standard damage tolerance initial flaw size assumptions, fatigue crack growth life was calculated for each case. The calculated lives for cracks growing in critical plane 4 of bulkhead 1 are shown in Figure 12. Similar data were generated for setions 1, 2, 3 and 5 and for bulkheads 2 and 3.



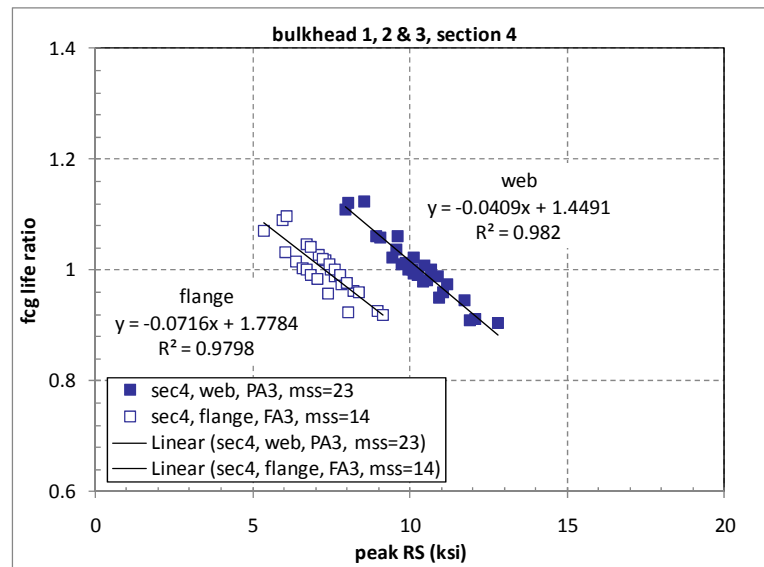
**Figure 12. Calculated Dependence of FCG Life on RS Variability Estimated by Scaling FEA Profiles for Bulkhead 1, Section 4.**

There are a number of useful interpretations of this data. To begin with, if we examine the dependence of FCG life on peak residual stress at a given location, we get results like the ones shown in Figure 13. These results provide insight regarding the dependence of fatigue crack growth life on stress magnitude for a given distribution shape. (Again, all FCG analyses used the same geometry / crack configuration.) As shown in Fig. 13, the results for both the web and flange of section 4 indicate a nearly linear relationship between peak residual stress and FCG life for the range of distributions studied. As shown, for the web, the slope of this linear relationship is -0.0511, which means that for each increase in the peak RS value of 1 ksi, there is a decrease in the life ratio of approximately 5% (so a 5 ksi increase in peak RS in the 8 to 14 ksi range at this location for this spectrum results in approximately a 25% decrement in life). The slope for the flange is slightly higher, -0.0561, indicating a slightly larger life decrement, approximately 28%, for each 5 ksi increase in peak RS in the 6 to 9 ksi range. This ‘RS sensitivity’ was evaluated for each critical plane of each bulkhead.



**Figure 13. Calculated Dependence of FCG Life Ratios on Peak RS (RS Sensitivity) for Bulkhead 1, Section 4.**

If we group the results for bulkheads 1, 2 and 3 together, then we find the scatter in RS sensitivity due to machined part placement within the forging. These results are shown for critical plane 4 in Figure 14. The RS sensitivity results for all critical planes are given in Table 2. Note, the higher the slope, the greater the sensitivity of FCG life on peak RS.

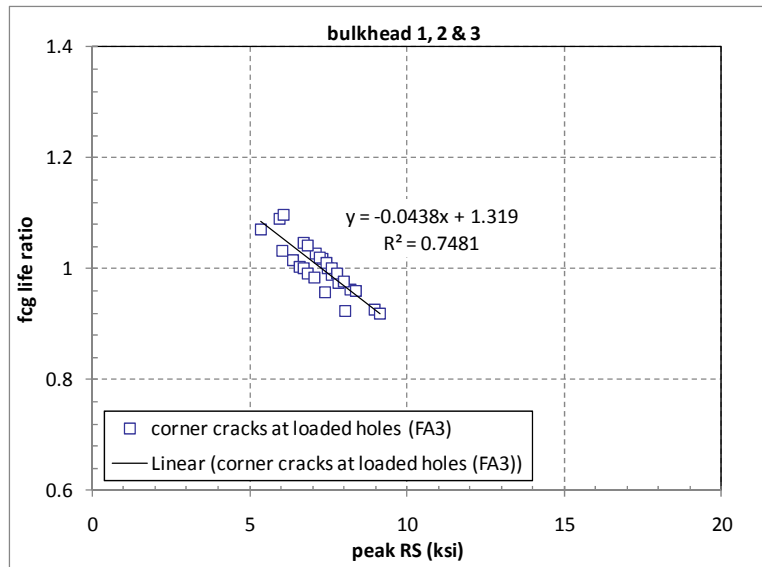


**Figure 14. Calculated Dependence of FCG Life Ratios on Peak RS (RS Sensitivity) for Bulkheads 1, 2 & 3, Section 4.**

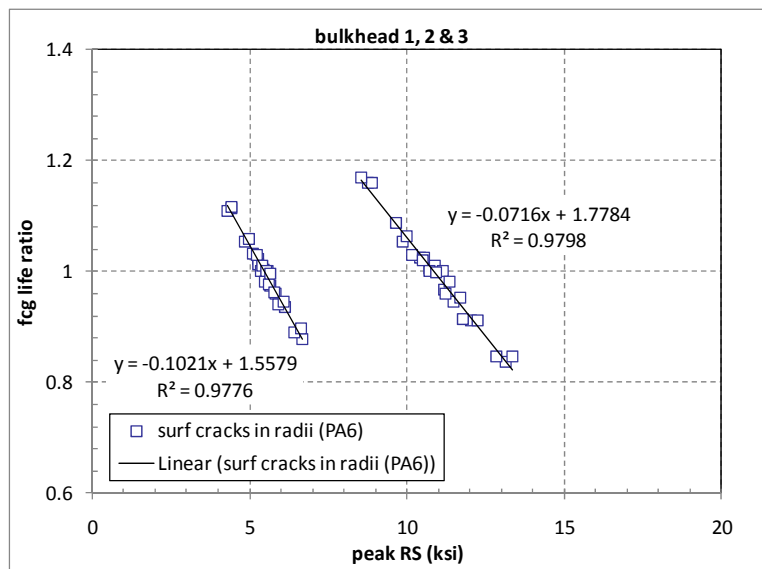
**Table 2. Calculated FCG Life Sensitivity (to peak residual stress) Based on Location.**  
slope of life ratio vs. peak stress relationship

location	bh1	bh2	bh3	avg	$\Delta\text{life}(\%)/\text{ksi}$
section 1, flange	-0.0357	-0.0323	-0.0504	-0.03947	3.9
section 1, web	-0.0397	-0.0265	-0.0311	-0.03243	3.2
section 2, flange	-0.0418	-0.0298	-0.0569	-0.04283	4.3
section 2, web	-0.059	-0.0619	-0.0592	-0.06003	6.0
section 3, flange	-0.0723	-0.0697	-0.0761	-0.0727	7.3
section 3, web	-0.0412	-0.0399	-0.0436	-0.04157	4.2
section 4, flange	-0.0561	-0.0578	-0.0546	-0.05617	5.6
section 4, web	-0.0511	-0.0518	-0.0511	-0.05133	5.1
section 5, flange	-0.1068	-0.1036	-0.1008	-0.10373	10.4
section 5, web	-0.1241	-0.133	-0.1349	-0.13067	13.1

RS sensitivity results can also be grouped according to fracture model; such data could illustrate the degree to which the sensitivity is dependent on the type of geometry / crack. The results for corner cracks at loaded fastener holes and surface cracks in radii are shown in Figures 15 and 16 respectively, and the results for all of the configurations addressed in this study are summarized in Table 3.



**Figure 15. Calculated Dependence of FCG Life Ratios on Peak RS (RS Sensitivity) for Corner Cracks at Loaded Fastener Holes (FA3).**



**Figure 47. Calculated Dependence of FCG Life Ratios on Peak RS (RS Sensitivity) for Semi-Elliptical Surface Cracks in Radii (PA6), Bulkheads 1, 2 & 3.**

## VI. Conclusions

A detailed description of the computed variability in both the magnitude and spatial distribution of residual stresses in a representative forging has been given. The manner in which these stresses can be used to calculate residual stress intensity factors and thereby influence fatigue crack growth life has been discussed. Finally, the dependence of fatigue life variability on the types of magnitude and spatial variability in residual stress found by FEA simulation have been quantified. For the conditions considered in this study (forging process, machined part placement, critical location / crack geometry, material and fatigue spectrum) we have found that variability in fatigue crack growth life due to part placement is on the order of +/- 20% about the mean. Furthermore, we have found from scaling the RS distributions for these conditions, that fatigue crack growth life sensitivity is on the order of 15% to 50% change in FCG life with every 5ksi change in peak RS.

**Table 3. Calculated FCG Life Sensitivity (to peak residual stress) Based on Crack Type.**

crack type	location	slope	$\Delta\text{life}(\%)/\text{ksi}$
FA1: thru crack at loaded hole	sec1, flange, bh1	-0.0357	4.1
	sec1, flange, bh2	-0.0323	
	sec1, flange, bh3	-0.0504	
	sec2, flange, bh1	-0.0418	
	sec2, flange, bh2	-0.0298	
	sec2, flange, bh3	-0.0569	
	avg.	-0.04115	
FA3: qtr-elliptical corner crack at loaded hole	sec4, flange, bh1	-0.0561	5.6
	sec4, flange, bh2	-0.0578	
	sec4, flange, bh3	-0.0546	
	avg.	-0.05617	
HA1: thru crack at open hole	sec5, web, bh1	-0.1241	13.1
	sec5, web, bh2	-0.133	
	sec5, web, bh3	-0.1349	
	avg.	-0.13067	
PA3: thru crack in radius	sec1, web, bh1	-0.0397	4.6
	sec1, web, bh2	-0.0265	
	sec1, web, bh3	-0.0311	
	sec2, web, bh1	-0.059	
	sec2, web, bh2	-0.0619	
	sec2, web, bh3	-0.0592	
	sec3, web, bh1	-0.0412	
	sec3, web, bh2	-0.0399	
	sec3, web, bh3	-0.0436	
	sec4, web, bh1	-0.0511	
	sec4, web, bh2	-0.0518	
	sec4, web, bh3	-0.0511	
	avg.	-0.04634	
PA6: semi-elliptical surface crack in radius	sec3, flange, bh1	-0.0723	8.8
	sec3, flange, bh2	-0.0697	
	sec3, flange, bh3	-0.0761	
	sec5, flange, bh1	-0.1068	
	sec5, flange, bh2	-0.1036	
	sec5, flange, bh3	-0.1008	
	avg.	-0.08822	

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